

Lower bound for the ratio of charged wino tracks to charged lepton tracks at LHC.

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Abstract: By using kinematical arguments we derive a theorem for the ratio: [production cross-section of charged-neutral wino pairs plus anything else] over [production cross-section of charged-neutral lepton pairs plus anything else]. We do that by working out the consequences of substituting the lepton pair with a wino pair, leaving untouched everything else of the interaction. We apply this result to the possible production of winos at LHC obtaining a lower bound for the ratio of the number of charged-winos over charged-leptons tracks and also find the average charged-wino track length, within a region in the relevant parameter space (e.g. available energy and mass).

General framework.

One relevant hypothesis is that the dark matter particles are non-relativistic neutral winos, namely the neutral member of a triplet of (very) massive fermions in the adjoint representation of the weak $SU(2)_W$, and which can only interact with the $SU(2)_W$ vector bosons. This picture is per se worthwhile, even independently of the assumption of supersymmetry.

This hypothesis makes the dark matter very dark, because there is no tree-level elastic scattering of neutral winos on ordinary matter, because they are not coupled to the neutral vector boson, and the non relativistic inelastic scattering on ordinary matter giving neutral-wino \rightarrow charged-wino is not allowed even for a tiny charged-neutral mass difference of the order of tens of MeV . Therefore, the interaction of dark matter with ordinary matter can only occur at the order one-loop in the weak interactions.

This is a somewhat extreme hypothesis, but one can also relax it imagining that the neutralino is a linear combination of a wino and a higgsino (that is a member of the fundamental representation of $SU(2)_W$). This would make dark matter less dark as tree level elastic scattering is possible in this case. Actually, a study of the dark-matter relic density indicates that in the TeV mass-range dark matter should be predominantly pure wino [1].

We consider here the possible production of a pair made of a charged and a neutral wino, plus anything else. This process comes from the coupling of the pair to a virtual W^\pm vector boson (this is so even in the case of a mixing with higgsino, because its coupling to higgs is proportional to the weak coupling constant and we neglect the higgs coupling to the ordinary matter).

The key point is that the production of a charged(electron or muon)-neutral(neutrino) lepton (one of them being anti-lepton) pair, plus anything else, also comes from the coupling of the pair to a virtual W^\pm vector boson (neglecting the coupling to higgs). Therefore, one can work out the consequences of substituting that lepton pair with the wino pair of the same total energy ³, leaving everything else of the to be the same. In this way one avoids to make any assumption on the dynamics of the production of the vector boson and of any other accompanying processes.

To summarize, we do not attempt to find the cross-section for the production of winos, but only to find a lower bound for the ratio of the production rate of wino pairs to the rate of lepton pairs. In other words, our aim is to state that, if there are winos of a given mass m_w , the observation of a production rate of seemingly unpaired charged leptons with momentum q (the neutral lepton of the pair being unobserved) implies the observation of a rate of charged wino tracks larger than a bound depending on m_w/q . We have in mind the direct observation of those tracks, besides the observation of the products of the decay charged-wino \rightarrow neutral wino, and we also derive bounds for the lengths of these charged wino tracks.

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³we neglect radiative corrections of the pair final state

In particular, we can consider the possible production of winos at LHC. If the winos are quite massive the total energy of the pair will be quite high and since we compare their rate of production with the rate of charged-neutral lepton pairs of the same total energy, these lepton pairs will presumably come from a W-boson directly produced in the parton interaction (a process usually called DY), rather than from the secondary decay of other produced particles. However, even if that picture is rather plausible, we are not obliged to make such an assumption. Our results depend on the ratio of the wino mass to the lepton energy, and when this ratio is larger than 1 there is simply no possibility of wino production.

Main results.

We consider the relation among the production of a charged-neutral lepton-antilepton pair (say $l^\pm l^0 =$ either $e^- \bar{\nu}_e$, or $e^+ \nu_e$, or $\mu^- \bar{\nu}_\mu$, or $\mu^+ \nu_\mu$) and the production of a charged wino-neutral wino pair (either $w_{ino}^- w_{ino}^0$, or $w_{ino}^+ w_{ino}^0$). We take the center-of-mass frame of the pair and indicate by Q_0 the total (lepton or wino) pair energy in this frame.

Our main result, derived in the Appendix A, is that

$$R \equiv \frac{\sigma_{w_{ino}^\pm w_{ino}^0}}{\sigma_{l^\pm l^0}} \geq \hat{R} \equiv 4 \sqrt{1 - \frac{4m_w^2}{Q_0^2}} \quad (1)$$

where $\sigma_{w_{ino}^\pm w_{ino}^0}$ and $\sigma_{l^\pm l^0}$ are, respectively, the cross section⁴ for producing a charged-wino neutral-wino pair, plus anything else, and for producing a charged-lepton neutral-lepton pair, plus anything else, with the same squared invariant mass of the pair $= Q_0^2$. Here, we have assumed the approximation of considering massless leptons and taking the same mass m_w for the charged and neutral wino.

In the possible case of dark matter being a mixing of wino and higgsino of the same mass m_w : $\cos(\alpha)|wino\rangle + \sin(\alpha)|higgsino\rangle$, the only difference with the previous result is that the factor 4 should be replaced by $4\cos(\alpha)^2 + 2\sin(\alpha)^2$.

The ratio \hat{R} gives a lower bound for the expected number of charged wino tracks, once a number of charged lepton tracks has been observed.

This result is expressed in terms of quantities defined in the CM frame of the lepton pair, whereas the observations are made in the CM frame of the incident particles whose collision gives rise to the process in question, for instance the proton-proton CM frame in the case of LHC. One would then need to perform a Lorentz transformation parametrized by $\vec{\beta}$, the relative velocity of the two frames. In order to know $\vec{\beta}$ one should reconstruct the momentum of the neutrino partner of the observed charged lepton.

Nevertheless, we will see that one can obtain quite useful and stringent bounds on the possible wino observation even in the total ignorance of $\vec{\beta}$. Of course, by inserting further assumptions on the range of $\vec{\beta}$, motivated e.g. in the case of LHC by parton distributions and the like, those bounds tighten, however they do not significantly alter the information. In order to show that, one has to work out the kinematics relating the CM frame of the lepton pair and the CM frame of the incident particles.

By doing that, our result can be expressed in terms of the ratio $x \equiv \frac{m_w^2}{q^2}$, where q is the observed charged-lepton momentum in the CM frame of the incident particles, and of the so far unknown β and $z \equiv \frac{\vec{\beta} \cdot \vec{q}}{\beta q}$. From the computations of the Appendix B we get:

$$R \geq \hat{R} \equiv 4 \sqrt{1 - \frac{4m_w^2}{Q_0^2}} = 4 \sqrt{1 - \frac{m_w^2}{q^2} \frac{1 - \beta^2}{(1 - \beta z)^2}} \quad (2)$$

Really, if $\frac{4m_w^2}{Q_0^2} > 1$ there is no possible production of winos. Therefore a precise statement is

$$R \geq \hat{R}(x, \beta, z) \equiv 4 \text{RealPart} \sqrt{1 - x \frac{1 - \beta^2}{(1 - \beta z)^2}}, \quad \text{with } x \equiv \frac{m_w^2}{q^2} \quad (3)$$

⁴ we write σ for short, in the place of the more precise $\frac{d\sigma}{dQ_0^2}$, both for the winos and the leptons.

Assuming the forward -backward and cylindrical symmetry of the collision, $\vec{\beta}$ and $-\vec{\beta}$, and thus z and $-z$, are equally likely. Therefore, the average value of \hat{R} for a given $|z|$ is

$$\langle \hat{R} \rangle (x, \beta, |z|) = (\hat{R}(x, \beta, z) + \hat{R}(x, \beta, -z))/2 \quad (4)$$

In conclusion, from the observation of a number $N_l(q)$ of tracks of charged leptons with a momentum q measured in the CM frame of the collider (the neutral member of the pair being unobserved), we predict the minimal amount of charged wino tracks to be $\hat{R} \times N_l(q)$. In the following, by letting $\vec{\beta}$ vary in its range, we get, for a given m_w/q , a region of values for $\langle \hat{R} \rangle$.

Another interesting quantity is the expected length of the charged wino track. The dominant mode is the decay of the \pm charged wino into a π^\pm and the neutral wino and the lifetime of a charged wino at rest is [2]

$$\tau_0 = \frac{m_\pi m_\mu^2 (1 - \frac{m_\mu^2}{m_\pi^2})^2}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu_\mu) 16 \delta m^3 (1 - \frac{m_\pi^2}{\delta m^2})^{1/2}} \quad (5)$$

(where δm is the charged-neutral wino mass difference).

Therefore the mean track length of the charged wino is

$$L = \tau_0 \frac{\beta_w}{\sqrt{1 - \beta_w^2}} \quad (6)$$

where β_w is the velocity of the charged wino in the CM frame of the incident particles:

$\frac{\beta_w}{\sqrt{1 - \beta_w^2}} = \sqrt{\frac{q_{0w}^2}{m_w^2} - 1}$, q_{0w} being its energy in this frame. Also in this case, we need the information on $\vec{\beta}$ to compute q_{0w} .

The charged-neutral wino mass difference has been computed to be $\delta m = 160 MeV$ in refs.[3] [4] for wino mass of the order of TeV , and somewhat higher say $162 MeV$ including two loops radiative corrections in ref. [2], giving $c\tau_0 = 6.4 cm$ from eq.(5). From eq.(6) we get the mean track length (in units of centimeters)

$$L(x, \beta, z) = 6.4 \sqrt{\frac{q_{0w}^2}{m_w^2} (x, \beta, z) - 1} \quad (7)$$

where by the kinematical computations of Appendix B one can write $\frac{m_w^2}{q_{0w}^2}$ as a function of x, β, z .

For a given $|z|$ we expect to see on average a tract length

$$\langle L \rangle (x, \beta, |z|) = \frac{\hat{R}(x, \beta, z)L(x, \beta, z) + \hat{R}(x, \beta, -z)L(x, \beta, -z)}{\hat{R}(x, \beta, z) + \hat{R}(x, \beta, -z)} \quad (8)$$

In the figures 1) and 2) we show our results for $\langle \hat{R} \rangle$ and $\langle L \rangle$ for the pure wino case, for $m_w^2/q^2 = 0.5$ fig.1, and for $m_w^2/q^2 = 0.8$ fig.2.

In the left part of the figure we allow $\beta, |z|$ to vary in their full range $0 \leq \beta \leq 1$, $0 \leq |z| \leq 1$, for the sake of illustrating the variability of the results, even though $\beta \sim 1$ would not be reached at LHC energies for wino masses of the order of TeV .

In the right part of the figure we restrict the range ⁵ to $0 \leq \beta \leq 0.4$, $0 \leq |z| \leq \cos \pi/6$ (right part of the figure).

⁵in the Appendix C we discuss the possible ranges of β, z .

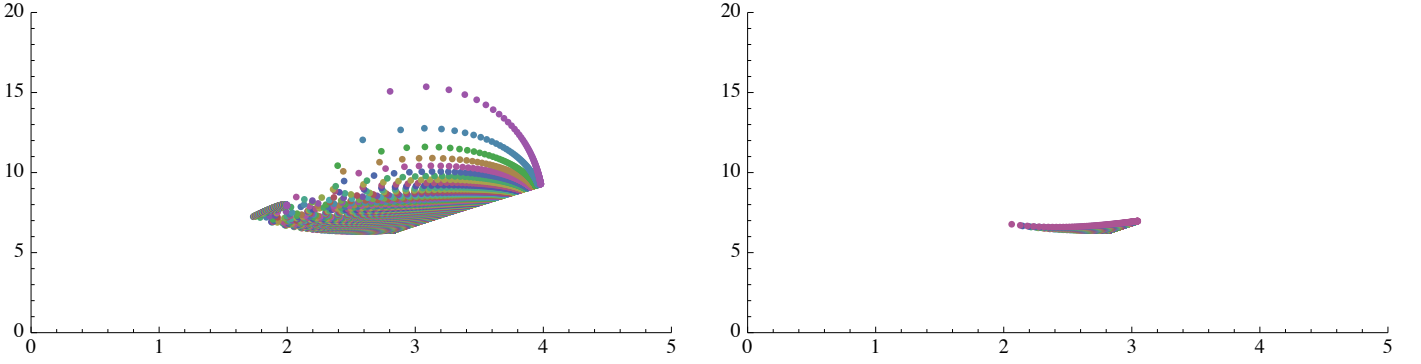


Figure 1: region of bounds for wino observation for $m_w^2/q^2 = 0.5$.
 Left: full range for $\beta, |z|$. Right: for $0 \leq \beta \leq 0.4$, $0 \leq |z| \leq \cos \pi/6$.
 x-axis $\langle \hat{R} \rangle$, y-axis $\langle L \rangle$ (in centimeters).

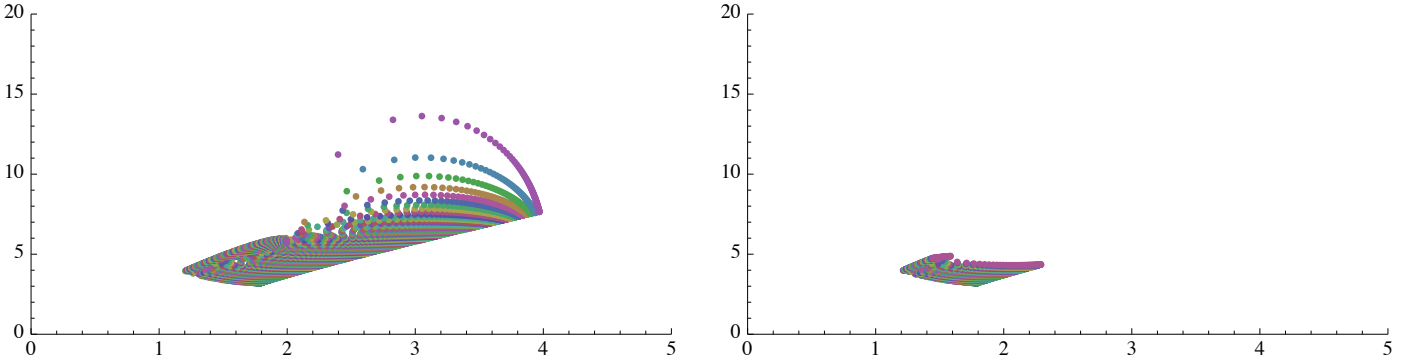


Figure 2: region of bounds for wino observation for $m_w^2/q^2 = 0.8$.
 Left: full range for $\beta, |z|$. Right: for $0 \leq \beta \leq 0.4$, $0 \leq |z| \leq \cos \pi/6$.
 x-axis $\langle \hat{R} \rangle$, y-axis $\langle L \rangle$ (in centimeters).

We can also compute the deviation of the expected track of the wino with respect to the direction of the observed track of the lepton. Namely, in terms of the differential cross-sections at a given polar angle in the LHC frame, the statement is:

$$d\sigma_{wino}(\theta'_{LHC}) \geq 4\sqrt{1 - \frac{m_w^2}{q^2} \frac{1 - \beta^2}{(1 - \beta z)^2}} d\sigma_{lepton}(\theta_{LHC}) \quad (9)$$

We want to get informations on the possible angular difference $\delta\theta_{LHC} = \theta'_{LHC} - \theta_{LHC}$ that is the angle difference of the directions of \vec{q}_w and \vec{q} . We can estimate this difference by referring the polar angles to the direction of $\vec{\beta}$. As we have seen, in the CM frame we take the same emission direction for the lepton and the wino, therefore z_{CM} is the same for both. But due to the mass difference of the lepton and the wino, the emission direction is different in the LHC frame, therefore z_w , defined by $\vec{q}_w \cdot \vec{\beta} = q_w \beta z_w$, is different from z , and we can work out its expression $z_w(x, \beta, z)$, see the Appendix B.

We can then compute the angle difference $\delta\theta_{LHC}$ of the direction of \vec{q}_w and \vec{q} :

$$|\delta\theta_{LHC}|_z = |\arccos[z] - \arccos[z_w(x, \beta, z)]| \quad (10)$$

Since, as we said, the sign of z is not defined, we take here too the average, like we did in defining $\langle L \rangle$:

$$\langle |\delta\theta_{LHC}| \rangle_{|z|} = \frac{\hat{R}_z |\delta\theta_{LHC}|_z + \hat{R}_{-z} |\delta\theta_{LHC}|_{-z}}{\hat{R}_z + \hat{R}_{-z}} \quad (11)$$

Our results for $\langle |\delta\theta_{LHC}| \rangle_{|z|}$ are reported in the following figures:

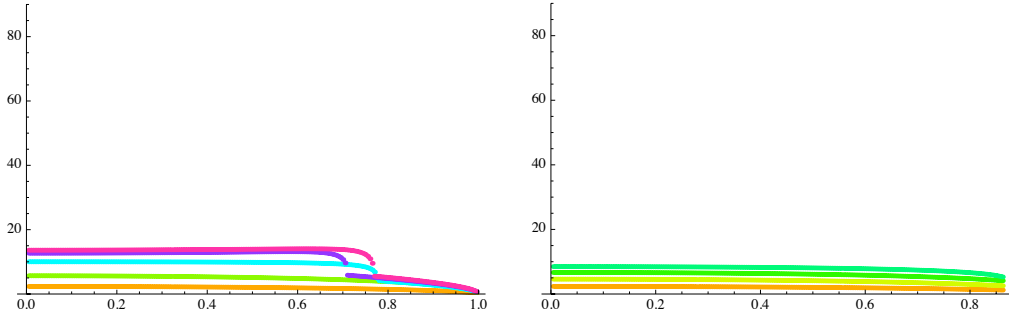


Figure 3: region of $\delta\theta_{LHC}$ for $m_w^2/q^2 = 0.5$ as a function of $|z|$ for various β (different colors).

Left: full range for $\beta, |z|$. Right: for $0 \leq \beta \leq 0.4$, $0 \leq |z| \leq \cos \pi/6$.

x-axis $|z|$, y-axis $< |\delta\theta_{LHC} = \theta_{wino} - \theta_{lepton}| >$ (in degrees).

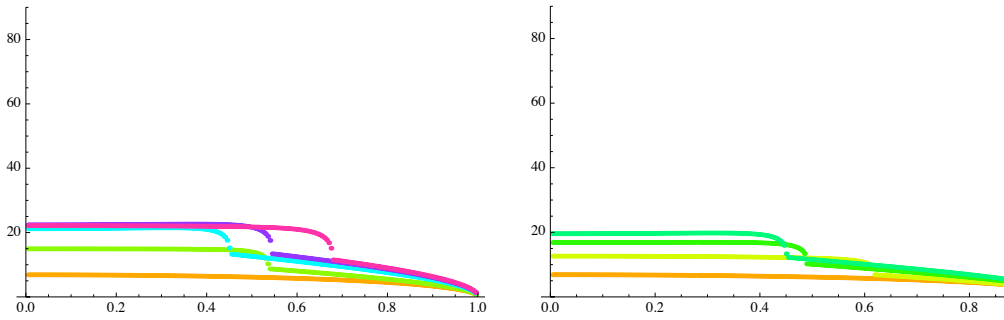


Figure 4: region of $\delta\theta_{LHC}$ for $m_w^2/q^2 = 0.8$ as a function of $|z|$ for various β (different colors).

Left: full range for $\beta, |z|$. Right: for $0 \leq \beta \leq 0.4$, $0 \leq |z| \leq \cos \pi/6$.

x-axis $|z|$, y-axis $< |\delta\theta_{LHC} = \theta_{wino} - \theta_{lepton}| >$ (in degrees).

Conclusions.

Our conclusion is that if seemingly unpaired charged leptons are observed at LHC, the neutral lepton of the pair being unobserved, (of course this would be a rare observation if their energy is very high, e.g. in the TeV range), then also tracks of charged weakly interacting particles in the adjoint representation of $SU(2)_W$, with mass less than the lepton momentum, should be observed, assuming they exist and have a lifetime comparable to what has been predicted in the literature.

We see from the figures that the number of charged wino tracks are expected to be from greater-or-equal-than up to four-times-as-much-as the number of lepton tracks, and their length of the order of various centimeters, from 2.5 up to 7, and higher for larger β , for wino-mass to lepton-momentum ratio in the range $0.7 - 0.9$ or less. Those charged wino tracks are expected to end giving rise to a charged lepton track, due to the weak decay charged wino \rightarrow neutral wino.

Otherwise, if no tracks of that kind are observed, our results can be used to put bounds on the existence of that kind of dark matter.

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Appendix A: derivation.

Consider first the lepton pair $l^\pm l^0$. Let us parametrize the momentum of the lepton in the lepton-pair CM frame by

$$q_{CM,\mu} = q_{CM}(1, \sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta)) \quad (12)$$

where θ, ϕ are angles with respect to arbitrary axes.

In this CM frame the chiral spinor, anti-spinor u^c, v^c representing the massless leptons are, in the representation in which γ_5 is diagonal: $\gamma_5 = \text{Diag}\{-1, -1, 1, 1\}$,

$$\begin{aligned} u^c(\vec{q}_{CM}) &= \sqrt{2q_{CM}}(e^{-i\phi} \sin(\theta/2), -\cos(\theta/2), 0, 0) \\ v^c(-\vec{q}_{CM}) &= \sqrt{2q_{CM}}(\cos(\theta/2), e^{i\phi} \sin(\theta/2), 0, 0) \end{aligned} \quad (13)$$

and the chiral lepton-pair current in this frame is

$$\begin{aligned} J_\mu^{lepton}(\theta, \phi) &= \bar{v}^c(-\vec{q}_{CM}) \gamma_\mu \frac{1 - \gamma_5}{2} u^c(\vec{q}_{CM}) \\ &= 2q_{CM} \left(0, \cos^2\left(\frac{\theta}{2}\right) - e^{-2i\phi} \sin^2\left(\frac{\theta}{2}\right), -i(\cos^2\left(\frac{\theta}{2}\right) + e^{-2i\phi} \sin^2\left(\frac{\theta}{2}\right)), -e^{-i\phi} \sin(\theta) \right) \end{aligned} \quad (14)$$

For reference, note also the anti-chiral spinor, anti-spinor u^a, v^a

$$\begin{aligned} u^a(\vec{q}_{CM}) &= \sqrt{2q_{CM}}(0, 0, \cos(\theta/2), e^{i\phi} \sin(\theta/2)) \\ v^a(-\vec{q}_{CM}) &= \sqrt{2q_{CM}}(0, 0, -e^{-i\phi} \sin(\theta/2), \cos(\theta/2)) \end{aligned} \quad (15)$$

and the corresponding anti-chiral current

$$J_\mu^a(\theta, \phi) = \bar{v}^a(-\vec{q}_{CM}) \gamma_\mu \frac{1 + \gamma_5}{2} u^a(\vec{q}_{CM}) = J_\mu^{lepton}(\theta, \phi)^* \quad (16)$$

Consider now the $w_{ino}^\pm w_{ino}^0$ pair (in the approximation in which the charged and the neutral wino have equal mass m_w) in the CM frame of the pair with the same total energy $Q_0 = 2q_{CM}$ as the leptons, taking the wino momentum \vec{q}_{wCM} parallel to the lepton one \vec{q}_{CM} , with $q_{wCM} = \sqrt{q_{CM}^2 - m_w^2}$. Their four-momentum will be:

$$q_{wCM, \mu} = q_{wCM} \left(\frac{q_{CM}}{q_{wCM}}, \sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta) \right) \quad (17)$$

One can choose the orthogonal basis of the helicity eigenstates for the spinors, anti-spinors representing the various possible states of the wino in this frame. In this basis, there are states which we call pseudo-chiral (pc) :

$$\begin{aligned} u^{pc}(\vec{q}_{wCM}) &= N_{upc} \left(e^{-i\phi} \sin \frac{\theta}{2}, -\cos \frac{\theta}{2}, \frac{-q_{wCM} + q_{CM}}{m_w} e^{-i\phi} \sin \frac{\theta}{2}, \frac{q_{wCM} - q_{CM}}{m_w} \cos \frac{\theta}{2} \right) \\ v^{pc}(-\vec{q}_{wCM}) &= N_{vpc} \left(\cos \frac{\theta}{2}, e^{i\phi} \sin \frac{\theta}{2}, \frac{q_{wCM} - q_{CM}}{m_w} \cos \frac{\theta}{2}, \frac{q_{wCM} - q_{CM}}{m_w} e^{i\phi} \sin \frac{\theta}{2} \right) \end{aligned} \quad (18)$$

and states which we call pseudo-antichiral (pa):

$$\begin{aligned} u^{pa}(\vec{q}_{wCM}) &= N_{upa} \left(\cos \frac{\theta}{2}, e^{i\phi} \sin \frac{\theta}{2}, \frac{q_{wCM} + q_{CM}}{m_w} \cos \frac{\theta}{2}, \frac{q_{wCM} + q_{CM}}{m_w} e^{i\phi} \sin \frac{\theta}{2} \right) \\ v^{pa}(-\vec{q}_{wCM}) &= N_{vpa} \left(e^{-i\phi} \sin \frac{\theta}{2}, -\cos \frac{\theta}{2}, \frac{-q_{wCM} - q_{CM}}{m_w} e^{-i\phi} \sin \frac{\theta}{2}, \frac{q_{wCM} + q_{CM}}{m_w} \cos \frac{\theta}{2} \right) \end{aligned} \quad (19)$$

with $N_{upc}^2 = N_{vpc}^2 = \frac{m_w^2}{q_{CM} - q_{wCM}}$ and $N_{upa}^2 = N_{vpa}^2 = \frac{m_w^2}{q_{CM} + q_{wCM}}$. Note that $u^{pc, pa} \rightarrow u^{c, a}, v^{pc, pa} \rightarrow v^{c, a}$ in the limit $q_{CM} \rightarrow \infty$.

In this basis we get four possible currents namely $J_\mu^{pc, pc}, J_\mu^{pa, pa}, J_\mu^{pc, pa}, J_\mu^{pa, pc}$, where $J_\mu^{i, j} = \bar{v}^i \gamma_\mu u^j$ (the winos have pure vectorial coupling to the $SU(2)_W$ gauge bosons).

The key point of the derivation is that

$$\begin{aligned} J_\mu^{pc, pc}(\theta, \phi) &= 2q_{CM} \left(0, \cos^2\left(\frac{\theta}{2}\right) - e^{-2i\phi} \sin^2\left(\frac{\theta}{2}\right), -i(\cos^2\left(\frac{\theta}{2}\right) + e^{-2i\phi} \sin^2\left(\frac{\theta}{2}\right)), -e^{-i\phi} \sin(\theta) \right) = J_\mu^{lepton}(\theta, \phi) \\ J_\mu^{pa, pa}(\theta, \phi) &= 2q_{CM} \left(0, \cos^2\left(\frac{\theta}{2}\right) - e^{2i\phi} \sin^2\left(\frac{\theta}{2}\right), i(\cos^2\left(\frac{\theta}{2}\right) + e^{2i\phi} \sin^2\left(\frac{\theta}{2}\right)), -e^{i\phi} \sin(\theta) \right) = J_\mu^{lepton}(\theta, \phi)^* \end{aligned} \quad (20)$$

Since the cross-section is the sum of the square modulus of the amplitudes corresponding to the different currents, the cross-section for producing the wino pair will be larger than what one gets by taking the charged and the neutral wino to be both pc or both pa , that is by restricting the wino-currents to $J_\mu^{pc,pc}, J_\mu^{pa,pa}$:

$$\sigma_{wino} = \sigma(pc, pc) + \sigma(pa, pa) + \sigma(pc, pa) + \sigma(pa, pc) \geq \sigma(pc, pc) + \sigma(pa, pa) \quad (21)$$

One can express the cross-section in a Lorentz invariant way by integrating the modulus square of a relativistic invariant amplitude over the relativistic invariant final particles phase space (and dividing by a Lorentz invariant factor representing the incident flux times the initial particles' density). Therefore, one can evaluate the cross-section for producing the lepton pair by performing the final phase space integration in the CM frame of the pair at a fixed Q_0^2 (here and in the following we write σ for short in the place of $d\sigma/dQ_0^2$) and write

$$\sigma_{lepton} = \int |K^\mu J_\mu^{lepton}(\theta, \phi)|^2 d\phi \sin \theta d\theta \quad (22)$$

whereas for the cross-section for producing the wino pair one has

$$\sigma(pc, pc) = \sigma(pa, pa) = 2 \int |K^\mu J_\mu^{lepton}(\theta, \phi)|^2 d\phi \sin \theta d\theta \sqrt{1 - \frac{4m_w^2}{Q_0^2}} \quad (23)$$

with the same K^μ , whatever it is, the factor 2 coming from the fact that the lepton vertex is $\frac{g}{\sqrt{2}} \bar{\nu} \gamma_\mu \frac{1-\gamma_5}{2} u$ and the wino vertex is $g \bar{\nu} \gamma_\mu u$. One can check by explicit integration that

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta |K^\mu J_\mu^{pc,pc}(\theta, \phi)|^2 = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta |K^\mu J_\mu^{pa,pa}(\theta, \phi)|^2 = \frac{32\pi}{3} q_{CM}^2 (|K_1|^2 + |K_2|^2 + |K_3|^2) \quad (24)$$

from which the equality $\sigma(pc, pc) = \sigma(pa, pa)$ follows. Therefore

$$\sigma(pc, pc) + \sigma(pa, pa) = 4 \int |K^\mu J_\mu^{lepton}|^2 d\phi \sin \theta d\theta \sqrt{1 - \frac{4m_w^2}{Q_0^2}} = 4\sigma_{lepton} \sqrt{1 - \frac{4m_w^2}{Q_0^2}} \quad (25)$$

and finally

$$\sigma_{wino} \geq \sigma(pc, pc) + \sigma(pa, pa) = 4 \sqrt{1 - \frac{4m_w^2}{Q_0^2}} \sigma_{lepton} \quad (26)$$

Therefore

$$R \equiv \frac{\sigma_{wino}}{\sigma_{lepton}} \geq \hat{R} \equiv 4 \sqrt{1 - \frac{4m_w^2}{Q_0^2}} \quad (27)$$

In the possible case of dark matter being a mixing of wino and higgsino of the same mass m_w : $\cos(\alpha)|wino\rangle + \sin(\alpha)|higgsino\rangle$, the only difference with the previous result is that the factor 4 should be replaced by $4\cos(\alpha)^2 + 2\sin(\alpha)^2$, since the higgsinos can be rearranged to form a Dirac doublet vectorially coupled to the W -boson with coupling $g/\sqrt{2}$ (we can neglect the higgs contribution, since the higgs boson coupling to the higgsino is also proportional to g and the higgs coupling to the incoming matter matter is exceedingly small).

Appendix B: kinematical computations.

We call \vec{q} the three-momentum of the observed charged lepton and q_0 its energy in the CM frame of the incident particles. For the lepton, in the massless approximation, $q_0 = q$. We call \vec{q}_w the wino three-momentum and q_{0w} its energy in the CM frame of the incident particles, with $q_w = \sqrt{q_{0w}^2 - m_w^2}$.

By calling $\vec{\beta}$ the velocity of the CM pair frame with respect to the frame and defining z by

$$\vec{q} \cdot \vec{\beta} = q\beta z \quad (28)$$

we have

$$q_{CM} = \gamma(q - \vec{\beta} \cdot \vec{q}) = q \frac{1 - \beta z}{\sqrt{1 - \beta^2}} \quad (29)$$

From that we get:

$$\frac{m_w^2}{q_{CM}^2} = \frac{m_w^2}{q^2} \frac{1 - \beta^2}{(1 - \beta z)^2} \quad (30)$$

As for β_w (the velocity of the charged wino in the LHC frame to be used for determining the track length), it can be obtained from the Lorentz transformations:

$$\frac{q_{0w}}{m_w} = \frac{\frac{q_{CM}}{m_w} + \frac{\vec{\beta} \cdot \vec{q}_{CM}}{q_{CM}} \sqrt{\frac{q_{CM}^2}{m_w^2} - 1}}{\sqrt{1 - \beta^2}}, \quad \frac{q_{CM}}{m_w} = \frac{1}{\sqrt{x}} \frac{1 - \beta z}{\sqrt{1 - \beta^2}}, \quad \frac{\vec{\beta} \cdot \vec{q}_{CM}}{q_{CM}} = \beta \frac{z - \beta}{1 - \beta z} \quad (31)$$

(Note: $\frac{q_{0w}}{m_w}$ is minimal for $\frac{\vec{\beta} \cdot \vec{q}_{CM}}{q_{CM}} = -\beta$ and $\frac{q_{CM}}{m_w} = \frac{1}{\sqrt{1 - \beta^2}}$, where it is $= 1$). From eq.(31) we get $\frac{q_{0w}^2}{m_w^2}$ as a function of x, β, z :

$$\frac{q_{0w}^2}{m_w^2}(x, \beta, z) = \frac{[(1 - \beta z)^2 - \beta(\beta - z)\sqrt{(1 - \beta z)^2 - (1 - \beta^2)x}]^2}{(1 - \beta^2)^2(1 - \beta z)^2 x} \quad (32)$$

Next, by defining z_{CM} by $\vec{q}_{CM} \cdot \vec{\beta} = q_{CM}\beta z_{CM}$ we get from eq.(31)

$$z_{CM} = \frac{z - \beta}{1 - z\beta} \quad (33)$$

As we have seen, in the CM frame we take the same emission direction for the lepton and the wino, therefore z_{CM} is the same for both. But due to the mass difference of the lepton and the wino, the emission direction is different in the LHC frame, therefore z_w , defined by $\vec{q}_w \cdot \vec{\beta} = q_w\beta z_w$, is different from z . In fact, by the Lorentz transformation

$$\begin{aligned} \vec{\beta} \cdot \vec{q}_w &= \beta q_w z_w = \frac{\vec{\beta} \cdot \vec{q}_{wCM} + \beta^2 q_{CM}}{\sqrt{1 - \beta^2}} = \frac{\beta q_{wCM} z_{CM} + \beta^2 q_{CM}}{\sqrt{1 - \beta^2}} \\ \rightarrow z_w(x, \beta, z) &= \left(\frac{\sqrt{q_{CM}^2/m_w^2 - 1}}{\sqrt{q_{0w}^2/m_w^2 - 1}} z_{CM} + \beta \frac{q_{CM}/m_w}{\sqrt{q_{0w}^2/m_w^2 - 1}} \right) \frac{1}{\sqrt{1 - \beta^2}} \end{aligned} \quad (34)$$

and from that we can work out the expression $z_w(x, \beta, z)$.

Appendix C: possible ranges for β, z .

We have seen that, even though $\beta, |z|$ are not known, we can find a region of values for $< \hat{R} >, < L >, < \delta\theta_{LCH} >$ for a given x , by varying $\beta, |z|$ in their full range: $0 \leq \beta \leq 1, 0 \leq |z| \leq 1$. It can be nevertheless worthwhile to discuss which can be the most likely ranges for $\beta, |z|$. Here we take the case of LHC and we assume that the relevant process is DY.

Let us discuss the expected magnitude of β and its direction. It can originate from 1) mismatch of the almost opposite incoming parton momenta and/or 2) the momentum of the produced vector-boson.

In the case 1) the direction of $\vec{\beta}$ is essentially the LHC beam direction. Let us call $\zeta = \hat{s}/s$ where s is the nominal squared energy of the improved LHC, say $s = (14 \text{ TeV})^2$, and \hat{s} is the squared energy of the colliding partons. We have $\zeta = xy$ where x, y are the momentum fraction carried by the colliding partons, and the velocity of the colliding parton frame is $\beta = \frac{x-y}{x+y} = \frac{\zeta-y^2}{\zeta+y^2}$. We are interested in the case where ζ is sizable, say $\zeta \geq 0.6$. On average, we get (from the code [6]) $\beta \sim 0.19$ for $\zeta = 0.6$, $\beta \sim 0.08$ for $\zeta = 0.8$.

In the case 2) the maximum value of β is reached when in the final state there is the virtual vector boson (producing the lepton or wino pair) and a massless object. Actually, the dominant process is the collision of a gluon with a quark producing a vector-boson and a quark. By taking the quark as

massless, this process is enhanced by a would-be collinear divergence of the diagram having a pole in the Mandelstam u -variable ⁶, when the vector-boson is anti-parallel to the direction of the incoming quark, that is the beam direction. This being so, in both cases 1) and 2) $\vec{\beta}$ can be taken along the beam direction.

As for its magnitude, $\beta = \frac{\hat{s}-Q_0^2}{\hat{s}+Q_0^2}$. Remembering from Appendix B that $Q_0^2 = 4q^2(1-\beta z)^2/(1-\beta^2)$ we get $\beta_{MAX} \sim 0.55$ for $4q^2/\hat{s} = 0.8$ and $\beta_{MAX} \sim 0.35$ or $4q^2/\hat{s} = 0.9$.

From those considerations, we have taken $0 \leq \beta \leq 0.4$ as a reasonable range.

As for z , that is the cosine of the angle θ_{lepton} of the observed lepton with the direction of $\vec{\beta}$, assuming this direction is the one of the LHC beam, we may imagine to limit the observation to tracks which are outside a cone around the beam axis and limit that angle to be $-\pi/6 \leq \theta_{lepton} \leq \pi/6$.

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⁶The process gluon-gluon→vector boson *via* a quark loop would imply the same graph as the decay of a spin 1 particle into two massless vectors, which is forbidden ([5]).